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THE DYNAMIC PROBLEM OF ELECTROELASTICITY FOR A NON-HOMOGENEOUS CYLINDER†

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The non-stationary coupled problem of electroelasticity in connection with the dynamic twisting of a finite hollow cylinder of non-homogeneous piezoelectric material is considered in the case when the electric potential or shear stresses, which depend arbitrarily on time, are specified on its curvilinear surfaces. The method of expansion in eigen vector-valued functions in the form of a structural algorithm of finite integral transformations is used. It is shown that a closed solution can be obtained for a power law of the non-homogeneity of the electric, elastic and inertial characteristics of the material. The results obtained hold for crystals of tetragonal symmetry of class 422 and the hexagonal system of class 622.

THE PROBLEM of the integrability of the equations of the theory of the elasticity of non-homogeneous isotropic and transversely isotropic bodies has been investigated fairly completely for the case of static loading [1–3]. When non-stationary interaction occurs between force and electric fields the method of expansion in eigen vector-functions [4] is effective. Using it together with the method of finite differences one can obtain solutions of some special problems of dynamic electro-elasticity for homogeneous bodies [4–6].

1. Suppose a hollow circular finite cylinder in a cylindrical system of coordinates (r, θ, z) occupies the region Ω: $\{a\leqslant r\leqslant b,\ 0\leqslant\theta\leqslant 2\pi,\ 0\leqslant z\leqslant l\}$, is a linearly elastic anisotropic body, and is made of a non-homogeneous piezoelectric material whose physical-mechanical and electrical characteristics vary continuously along the radius r. We will consider the case when the ends of the cylinder $(z=0,\ l)$ are free from stresses and electric charges, while shear stresses $\sigma(z, t)$ and an electric potential $p(z, t)$ act on the inner and outer curvilinear surfaces $(r=a,\ b)$, respectively. This formulation generalizes the physically realizable boundary conditions, since only $p(z, t)$ or $\sigma(z, t)$ are in fact specified. Since the cylinder performs forced torsional oscillations, we will assume that at the initial instant of time $(t=0)$ we know the distribution of the tangential displacements $g_1(r, z)$ and their velocities $g_2(r, z)$. It should be noted that the components of the stress tensor and the vector of the displacements in this case are independent of the angular coordinate θ.

The mathematical model of this problem includes differential equations of the motion and the electrostatics of a continuous piezoelectric medium [4, 5, 7]

$$\frac{\partial}{\partial r}\tau_{r\theta}+\frac{\partial}{\partial z}\tau_{z\theta}+\frac{2}{r}\tau_{r\theta}-\rho\frac{\partial^2}{\partial t^2}v=0$$

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$$\frac{\partial}{\partial r} D_r + \frac{\partial}{\partial z} D_z + \frac{1}{r} D_r = 0; \quad u = w = 0 \tag{1.1}$$

which are related by the equations of state

$$\tau_{r\theta} = c_{66}\left(\frac{\partial}{\partial r} v - \frac{1}{r} v\right), \quad \tau_{z\theta} = c_{44} \frac{\partial}{\partial z} v + e_{14} \frac{\partial}{\partial r} \Phi$$

$$D_r = e_{14} \frac{\partial}{\partial z} v + \epsilon_{11} \frac{\partial}{\partial r} \Phi, \quad D_z = -\epsilon_{33} \frac{\partial}{\partial z} \Phi \tag{1.2}$$

and are supplemented by the boundary and initial conditions.

The following representation holds for the one-dimensional non-homogeneity of the mechanical and electrical characteristics of the medium considered

$$\rho = Sf(r), \ c_{44} = C_{44}f(r), \ c_{66} = C_{66}f(r), \ e_{14} = E_{14}f(r), \ \epsilon_{11} = E_{11}f(r),$$
$$\epsilon_{33} = E_{33}f(r) \tag{1.3}$$

In relations (1.1)–(1.3) $\tau_{r\theta}(r, z, t)$ and $\tau_{z\theta}(r, z, t)$ are the components of the mechanical-stress tensor, $v(r, z, t)$ is the tangential component of the displacement vector, $D_r(r, z, t)$ and $D_z(r, z, t)$ are the components of the electric-induction vector, $\Phi(r, z, t)$ is the electric potential, $c_{ii}(r)$ and $\rho(r)$ are the elastic characteristics and the density of the material $(i = 4, 6)$, $\epsilon_{kk}(r)$ is the permittivity $(k = 1, 3)$, $e_{14}(r)$ is the piezoelectric modulus of the cylinder, C_{ii}, S, E_{kk} and E_{14} are the corresponding physical-mechanical and piezoelectric characteristics of the uniform medium (the properties of these material constants are described in detail in [7]), and $f(r)$ is an arbitrary dimensionless continuously differentiable heterogeneity function.

By substituting (1.2) and (1.3) into (1.1) we obtain a system of equations of dynamic torsion of a heterogeneous piezoelectric cylinder when combined force and electric fields act on it, and also the boundary conditions

$$C_{66}\left[\frac{\partial^2}{\partial r^2} v + F(r)\frac{\partial}{\partial r} v - \frac{1}{r} F(r) v\right] + C_{44} \frac{\partial^2}{\partial z^2} v + E_{14} \frac{\partial^2}{\partial r \partial z} \Phi - S\ddot{v} = 0$$

$$E_{14} \frac{\partial}{\partial z}\left[\frac{\partial}{\partial r} v + F(r) v\right] - E_{11}\left[\frac{\partial^2}{\partial r^2} \Phi + F(r)\frac{\partial}{\partial r} \Phi\right] - E_{33} \frac{\partial^2}{\partial z^2} \Phi = 0 \tag{1.4}$$

$$C_{44} \frac{\partial}{\partial z} v + E_{14} \frac{\partial}{\partial r} \Phi = 0, \quad \Phi = 0, \quad z = 0, l \tag{1.5}$$

$$C_{66}\left(\frac{\partial}{\partial r} v - \frac{1}{r} v\right) = \sigma^*(z, t), \quad \Phi = 0, \quad r = a$$

$$\frac{\partial}{\partial r} v - \frac{1}{r} v = 0, \quad \Phi = p(z, t), \quad r = b \tag{1.6}$$

$$v = g_1(r, z), \quad v^{\bullet} = g_2(r, z), \quad t = 0, \quad F(r) = f'(r)/f(r) + r^{-1}, \quad \sigma^* = \sigma/f(a) \tag{1.7}$$

The prime denotes differentiation with respect to r, and the asterisk denotes differentiation with respect to t.

2. The initial-value problem (1.4)–(1.7) considered can be solved by the method of integral transform. We will first use the finite cosine and sine Fourier transforms with respect to the variable z, taking the boundary conditions (1.5) into account. The boundary-value problem in the transformants $v_c(r, n, t)$, $\Phi_s(r, n, t)$ obtained in transform space can then be reduced to standard form. To do this we introduce the representation

$$v_c(r,n,t)=h_c(r)\sigma_c(n,t)+V_c(r,n,t)$$
$$\Phi_s(r,n,t)=h_s(r)p_s(n,t)+\varphi_s(r,n,t);\ \{h_c(r),h_s(r)\}\in C_2[a,b] \tag{2.1}$$

By substituting (2.1) into the equations and boundary conditions, taking into account the relations

$$h_c''(r)+F(r)h_c'(r)-[r^{-1}F(r)+\epsilon^{-1}\alpha_n^2]\,h_c(r)=0$$
$$h_s''(r)+F(r)h_s'(r)-\chi^{-1}\alpha_n^2 h_s(r)=0 \tag{2.2}$$

$$h_c'(a)-a^{-1}h_c(a)=C_{66}^{-1},\quad h_s(a)=0,\quad h_c'(b)-b^{-1}h_c(b)=0,\quad h_s(b)=1 \tag{2.3}$$

we reduce the transformed boundary-value problem to the standard form

$$C_{66}\left[\frac{\partial^2 V_c}{\partial r^2}+F(r)\frac{\partial V_c}{\partial c}-\frac{1}{r}F(r)V_c\right]-C_{44}\alpha_n^2V_c+E_{14}\alpha_n\frac{\partial\varphi_s}{\partial r}-SV_c^{\bullet\bullet}=P_c(r,n,t)$$
$$E_{11}\left[\frac{\partial^2\varphi_s}{\partial r^2}+F(r)\frac{\partial\varphi_s}{\partial r}\right]-E_{33}\alpha_n^2\varphi_s+E_{14}\alpha_n\left[\frac{\partial V_c}{\partial r}+F(r)V_c\right]=Q_s(r,n,t) \tag{2.4}$$

$$\frac{\partial V_c}{\partial r}-\frac{1}{r}V_c=0,\quad \varphi_s=0,\quad r=a,b \tag{2.5}$$

$$V_c=G_{1c}(r,n),\quad V_c^{\bullet}=G_{2c}(r,n),\quad t=0 \tag{2.6}$$

Here

$$P_c(r,n,t)=Sh_c(r)\sigma_c^{\bullet\bullet}(n,t)-E_{14}\alpha_nh_s'(r)p_s(n,t),$$
$$Q_s(r,n,t)=-E_{14}\alpha_n\sigma_c(n,t)[h_c'(r)+F(r)h_c(r)] \tag{2.7}$$
$$\{V_c(r,n,t),\sigma_c(n,t),g_{1c}(r,n),g_{2c}(r,n)\}=\int_0^l\{v(r,z,t),\sigma^*(z,t),g_1(r,z),g_2(r,z)\}\cos\alpha_nz\,dz$$
$$\{\Phi_s(r,n,t),p_s(n,t)\}=\int_0^l\{\Phi(r,z,t),p(z,t)\}\sin\alpha_nz\,dz;\quad \alpha_n=n\pi l^{-1}$$
$$G_{1c}(r,n)=g_{1c}(r,n)-h_c(r)\sigma_c(n,0),\quad G_{2c}(r,n)=g_{2c}(r,n)-h_c(r)\sigma_c^{\bullet}(n,0)$$
$$\epsilon=C_{66}/C_{44},\quad \chi=E_{11}/E_{33}$$

We now apply the degenerate finite integral transformation with respect to the variable r to boundary-value problem (2.4)–(2.6), i.e. a transformation of the form [8]†

$$q(\lambda_{in},n,t)=\int_a^b m(r)V_c(r,n,t)K_1(\lambda_{in},r)dr \tag{2.8}$$

$$\{V_c(r,n,t),\varphi_s(r,n,t)\}=\sum_{i=1}^{\infty}q(\lambda_{in},n,t)\{K_1(\lambda_{in},r),K_2(\lambda_{in},r)\}\,\|K_{in}\|^{-2} \tag{2.9}$$
$$\|K_{in}\|^2=\int_a^b m(r)K_1^2(\lambda_{in},r)dr$$

Here λ_{in} $(i=1,2,\ldots)$ are positive parameters forming a denumerable set, and $\|K_{in}\|$ is the norm of the vector-valued function of the kernel of the degenerate transformation.

†See also SENITSKII Yu. E., Investigation of the elastic strain of structural components in the case of dynamic actions by the method of finite integral transformations. Izd. Sarat. Gos. Univ., Saratov, 1985.

For the system of Eqs (2.4) considered, the weighting function is defined by the following quadrature [9]

$$m(r) = \exp\left[\int F(r)\,dr\right] \tag{2.10}$$

A feature of the finite integral transformation introduced is the fact that its transform (2.8) and the inversion formula (2.9), represented in vector form, contain a different number of components of the vector function of the kernel $\mathbf{K}(\lambda_{in}, r)$. The expansions (2.9) hold when the following orthogonality relation is satisfied [4]

$$\int_a^b m(r)K_1(\lambda_{in}, r)K_1(\lambda_{jn}, r)\,dr = \delta_i^j \|K_{in}\|^2 \tag{2.11}$$

where δ_i^j is the Kronecker delta.

It was shown in [10] that when the transforms $q(\lambda_{in}, n, t)$ $(i = 1, 2, \ldots)$ are bounded, the uniqueness of the representations and the convergence of the expansions in the metric of space L_2 defined by the inversion formulae (2.9) is ensured.

Following the structural algorithm of the finite integral transformation method [8] we multiply the first equation of (2.4) and the initial conditions (2.6) by $m(r)K_1(\lambda_{in}, r)$, and the second equation by $m(r)K_2(\lambda_{in}, r)$, and we integrate over the interval $[a, b]$ and add. Then integrating by parts and satisfying the conditions

$$\{C_{66}m(r)[(\partial V_c/\partial r)K_1 - V_cK_1'] + E_{14}\alpha_n m(r)(\varphi_sK_1 - V_cK_2) + E_{11}m(r)[(\partial\varphi_s/\partial r)K_2 - \varphi_sK_2']\}\Big|_a^b = 0 \tag{2.12}$$

$$\int_b^a m(r)[V_cL(K_1, K_2) + \varphi_sM(K_1, K_2)]\,dr = -\lambda_{in}^2 C_{66}\int_a^b m(r)V_cK_1\,dr \tag{2.13}$$

the first of which is the bilinear form at the ends of the interval equated to zero, while the second is the operational property, we obtain a denumerable system of Cauchy problems for the transform $q(\lambda_{in}, n, t)$

$$q^{\cdot\cdot}(\lambda_{in}, n, t) + \omega_{in}^2 q(\lambda_{in}, n, t) = -S^{-1}N(\lambda_{in}, n, t), \quad i = 1, 2, \ldots \tag{2.14}$$

$$q(\lambda_{in}, n, 0) = G_1(\lambda_{in}, n), \quad q\;\;(\lambda_{in}, n, t)|_{t=0} = G_2(\lambda_{in}, n), \quad t = 0$$

Here ω_{in} are the angular frequencies of torsional oscillations of the cylinder

$$G_1(\lambda_{in}, n) = \int_a^b m(r)G_{1c}(r, n)K_1(\lambda_{in}, r)\,dr, \quad G_2(\lambda_{in}, n) = \int_a^b m(r)G_{2c}(r, n)K_1(\lambda_{in}, r)\,dr$$

$$N(\lambda_{in}, n, t) = \int_a^b m(r)[P_c(r, n, t)K_1(\lambda_{in}, r) + Q_s(r, n, t)K_2(\lambda_{in}, r)]\,dr, \quad \omega_{in} = \lambda_{in}(C_{66}/S)^{1/2}$$

$$L(K_1, K_2) = C_{66}[K_1'' + F(r)K_1' - r^{-1}F(r)K_1] - C_{44}\alpha_n^2K_1 - E_{14}\alpha_nK_2'$$

$$M(K_1, K_2) = E_{11}[K_2'' + F(r)K_2'] - E_{33}\alpha_n^2K_2 - E_{14}\alpha_n[K_1' + F(r)K_1]$$

From Eq. (2.14) we can determine the transform of the finite integral transformation

$$q(\lambda_{in}, n, t) = G_1(\lambda_{in}, n)\cos\omega_{in}t + \omega_{in}^{-1}G_2(\lambda_{in}, n)\sin\omega_{in}t - (S\omega_{in})^{-1}\int_0^t N(\lambda_{in}, n, \tau)\sin\omega_{in}(t-\tau)\,d\tau \tag{2.15}$$

3. We will now consider relations (2.12) and (2.13), which, together with the boundary conditions (2.5), enable us to formulate the homogeneous boundary-value problem for the components K_1 and K_2 of the kernel of the transformation. From (2.13) we obtain the system of equations

$$L(K_1, K_2) + \lambda_{in}^2 C_{66} K_1 = 0, \quad M(K_1, K_2) = 0 \tag{3.1}$$

Equations (2.12) and (2.5) lead to the corresponding conditions

$$K_1'(\lambda_{in}, r) - r^{-1} K_1(\lambda_{in}, r) = 0, \quad K_2(\lambda_{in}, r) = 0, \quad r = a, b \tag{3.2}$$

Note that the eigen functions of the boundary-value problem (3.1), (3.2) satisfy the orthogonality relationship (2.11) and, consequently, the boundary-value problem (3.1), (3.2) is self-adjoint.

We will consider the problem of the integrability of system (3.1), since this is connected with the possibility of constructing a closed solution of the problem in question. By differentiating the second equation of (3.1) and bearing in mind the operator equation

$$[K_k' + F(r)K_k]' = K_k'' + F(r)K_k' - r^{-1}F(r)K_k, \quad k = 1, 2 \tag{3.3}$$

we conclude that system (3.1) is equivalent to a fourth-order resolvant

$$\epsilon\chi\nabla_F^4 K_1(\lambda_{in}, r) - \alpha_n^2 B_{in} \nabla_F^2 K_1(\lambda_{in}, r) + \alpha_n^4 A_{in} K_1(\lambda_{in}, r) = 0 \tag{3.4}$$

$$\nabla_F^2 = \frac{d^2}{dr^2} + F(r)\frac{d}{dr} - r^{-1}F(r)$$

$$B_{in} = \chi A_{in} + \epsilon + \eta, \quad A_{in} = 1 - \gamma_h^{-2}\lambda_{in}^2, \quad \gamma_h^2 = \alpha_n^2 \epsilon^{-1}, \quad \eta = E_{14}^2 (E_{33} C_{44})^{-1} \tag{3.5}$$

We introduce the generating equation

$$\nabla_F^2 K_1(\lambda_{in}, r) = -\xi_{in}^2 K_1(\lambda_{in}, r) \tag{3.6}$$

and from (3.4) we find

$$(\xi_{in}^2)_{1,2} = \alpha_n^2 [-B_{in} \pm (2\chi\epsilon)^{-1} (B_{in}^2 - 4\chi\epsilon A_{in})^{1/2}]$$

Reverting to Eqs (3.3) and (1.8), we obtain

$$F(r) = (m+1)r^{-1}, \quad f(r) = r^m \tag{3.7}$$

Here m is an arbitrary real constant.

Taking expressions (3.7) into account and making the replacement of variables in accordance with the formulae

$$K_{1N}(\lambda_{in}, r) = r^{-m/2} W_N(x), \quad x = \xi_{in} r, \quad N = 1, 2. \tag{3.8}$$

we can reduce (3.6) to a Bessel equation in $W_N(x)$.

If we take into account the linearity of differential equation (3.4) and also relation (3.8), its general solution can now be represented as follows:

$$K_1(\lambda_{in}, r) = \sum_{N=1}^{2} K_{1N}(\lambda_{in}, r) = r^{-m/2} \sum_{N=1}^{2} [A_{inN} J_{m/2+1}(\xi_{inN} r) +$$

$$+ B_{inN} Y_{m/2+1}(\xi_{inN} r)] \tag{3.9}$$

where $J_{m/2+1}(\cdot\cdot\cdot)$ and $Y_{m/2+1}(\cdot\cdot\cdot)$ are cylindrical functions of the first and second kind, and A_{inN}, B_{inN} are arbitrary constants of integration.

Expanding relations (3.6) and (3.9) we can determine from the first equation of system (3.1) the second component of the kernel of the transform

$$K_2(\lambda_{in}, r) = \frac{C_{44}}{E_{14}\alpha_n} r^{-m/2} \sum_{N=1}^{2} \beta_{inN} \xi_{inN}^{-1} [A_{inN} J_{m/2}(\xi_{inN} r) + B_{inN} Y_{m/2}(\xi_{inN} r)] \tag{3.10}$$

$$\beta_{inN} = \epsilon \xi_{inN}^2 + \alpha_n^2 A_{inN}, \quad N = 1, 2$$

The weighting function of the finite integral transform can be calculated from (2.10) and (3.7)

$$m(r) = r^{m+1} \tag{3.11}$$

while the square of the norm can be calculated from expressions (2.9) and (3.9)–(3.11).

By substituting expressions (3.9) and (3.10) into (3.2) we can form a homogeneous system of algebraic equations in A_{inN}, B_{inN}. From the condition for the solution to be non-trivial we obtain a transcendental equation for determining the eigen values λ_{in}, and we can find A_{inN}, B_{inN}

$$D(\lambda_{in}) = \det a_{sk} = 0, \quad s, k = 1, 2, 3, 4 \tag{3.12}$$

$$B_{in2} = D_4 = \det a_{sk}, \quad s, k = 1, 2, 3, \quad A_{inN} = D_N, \quad B_{in1} = D_3 \tag{3.13}$$

The determinants D_1, D_2 and D_3 follow from D_4 by replacing the first, second and third columns respectively by colon$\{a_{14}\ a_{24}\ a_{34}\}$.

In (3.12) and (3.13) we have introduced the following notation

$$a_{sk} = \beta_{ink} \xi_{ink}^{-1} J_{m/2}(\xi_{ink} r), \quad s, k = 1, 2, \quad r = a, b \text{ for } s = 1, 2$$

$$a_{sk} = \beta_{in,k-2} \xi_{in,k-2}^{-1} Y_{m/2}(\xi_{in,k-2} r), \quad s = 1, 2, \quad k = 3, 4, \quad r = a, b \text{ for } s = 1, 2$$

$$a_{sk} = \xi_{ink} J_{m/2+2}(\xi_{ink} r), \quad s = 3,4, \quad k = 1, 2, \quad r = a, b \text{ for } s = 3, 4$$

$$a_{sk} = \xi_{in,k-2} Y_{m/2+2}(\xi_{in,k-2} r), \quad s, k = 3, 4, \quad r = a, b \text{ for } s = 3, 4$$

4. The concluding stage of the investigation is the determination of the functions $h_c(r)$, $h_s(r)$ which occur in (2.1). We will use Eqs (2.2) and (3.7). Their general integrals can be written in terms of modified Bessel functions $I_\nu(\cdot\cdot\cdot)$, $K_\nu(\cdot\cdot\cdot)$. Taking boundary conditions (2.3) into account we obtain

$$h_c(r) = \frac{1}{\gamma_n C_{66}} \left(\frac{a}{r}\right)^{m/2} \frac{K_{m/2+2}(\gamma_n b) I_{m/2+1}(\gamma_n r) + I_{m/2+2}(\gamma_n b) K_{m/2+1}(\gamma_n r)}{I_{m/2+2}(\gamma_n a) K_{m/2+2}(\gamma_n b) - I_{m/2+2}(\gamma_n b) K_{m/2+2}(\gamma_n a)} \tag{4.1}$$

$$h_s(r) = \left(\frac{b}{r}\right)^{m/2} \frac{I_0(\mu_n a) K_0(\mu_n r) - K_0(\mu_n a) I_0(\mu_n r)}{I_0(\mu_n a) K_0(\mu_n b) - I_0(\mu_n b) K_0(\mu_n a)}$$

$$\mu_n = m^2/4 + \zeta^2, \quad \zeta^2 = \alpha_n^2 \chi^{-1}$$

Applying the formulae for the inversion of a degenerate finite integral transform (2.9) and a finite Fourier transform successively to expressions (2.15) and (2.1) we obtain expressions for the functions of the tangential displacements and the electric potential of the cylinder

$$v(r, z, t) = \sum_{n=0}^{\infty} \Omega_n^{-1} \cos\alpha_n z [h_c(r) \sigma_c(n, t) + \sum_{i=1}^{\infty} q(\lambda_{in}, n, t) K_1(\lambda_{in} r) \| K_{in} \|^{-2}] \tag{4.2}$$

$$\Phi(r, z, t) = \sum_{n=1}^{\infty} \frac{2}{l} \sin\alpha_n z \left[h_s(r)p_s(n, t) + \sum_{i=1}^{\infty} q(\lambda_{in}, n, t) K_2(\lambda_{in} r) \| K_{in} \|^{-2}\right]$$

$$\Omega_n = \begin{cases} l/2 & \text{for} \quad n \neq 0 \\ l & \text{for} \quad n = 0 \end{cases}$$

Equations (4.2) satisfy the differential equations (1.4) and the boundary conditions (1.5)–(1.7), i.e. they represent a closed solution of the problem for a power law (3.7) of the change along the radius of the physical-mechanical and electrical characteristics of the cylinder. Expressions (4.2) were constructed for arbitrary actions, and hence, taking different functional relationships as $\sigma(z, t)$, $p(z, t)$ and calculating the transformants (2.7) and (2.15), the corresponding particular results can be obtained. In the case when $m = 0$, solution (4.2) holds for a uniform piezoelectric cylinder. It should be noted that Eqs (1.4) for $m = 0$, $F(r) = r^{-1}$ represent (3.43) and (3.44) of [7] written in cylindrical coordinates for the case of antiplane strain, and crystals of classes 422 and 622 [7, Table 3.4].

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